Fano 3-folds, Lagrangian fibration and Leech-K3 geometry

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Prime Fano 3-folds $X_{2g-2} \subset \mathbb{P}^{g+1}$ exist for $g \leq 10$ and $g = 12$. They provide us lots of explicit geometries of homogeneous varieties, K3 surfaces and their higher dimensional analogue, that is, holomorphic symplectic varieties. I explain them with focus on Lagrangian fibrations. After a brief explanation of a pyramid of homogeneous varieties related with Fano's, I proceed to Hilbert squares, cubes of K3's etc..

1. Homogeneous contact manifolds and Hilbert square $S^{[2]}_{18}$ of K3 surfaces of genus 10
2. Homogeneous Legendrian manifolds and Hilbert cube $S^{[3]}_{16}$ of K3 surfaces of genus 9
3. Cubic 4-folds and conjectural OG10's with Conway configuration of (-2)-divisors