# Workshop on Moduli, K-stability, Fano varieties, and Related Topics 

IBS Center for Complex Geometry

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#### Abstract

Speaker: Shigeru Mukai (RIMS, Kyoto University) Title: Moduli of curves of genus 11, and prime Fano 3-folds of adjacent poristic genera

Abstact: Let $\mathcal{K}_{g} \subset \mathcal{M}_{g}$ be the locus of curves of genus $g \geq 2$ which are embeddable in a K3 surface. $\mathcal{K}_{g}$ is of expected dimension $\min \{3 g-3, g+19\}$ except for $g=10,12$, and one less for $g=10,12$. This implies the uniruledness of $\mathcal{M}_{11}$ for $g=11$ (Mori-M. 1983). If a curve $C$ in $\mathcal{K}_{10}$ does not belong to the BN-subloci $[2 \times 6]$ or $[3 \times 4]$, then $C$ is a (complete) linear section of the homogeneous contact variety of type $G_{2}$. In the case of genus 12 , the 30 -dimensional $\mathcal{K}_{12}$ contains four subloci [Clebsch-Lüroth], $[2 \times 7],[3 \times 5]$ and $[4 \times 4]$, corresponding to four onenodal degenerations of prime Fano 3 -folds $V_{22}$. If a curve $C$ in $\mathcal{K}_{12}$ belong to none of these loci, then $C$ is the intersection of two anti-canonical members of $V_{22}$ in a unique way (the last linear section theorem).


## Speaker: Kento Fujita (Osaka University)

Title: The Calabi problem for Fano threefolds
Abstact: There are 105 irreducible families of smooth Fano threefolds, which have been classified by Iskovskikh, Mori and Mukai. For each family, we determine whether its general member admits a Kaehler-Einstein metric or not. This is a joint work with Carolina Araujo, Ana-Maria Castravet, Ivan Cheltsov, Anne-Sophie Kaloghiros, Jesus Martinez-Garcia, Constantin Shramov, Hendrik Suess and Nivedita Viswanathan.

Speaker: Yuri Prokhorov (Steklov Mathematical Institute)
Title: On the classification of singular Fano threefolds
Abstact: I will give an overview of the current state of the problem of classification of singular Fano threefolds.

Speaker: Constantin Shramov (Steklov Mathematical Institute)

Title: Conic bundles
Abstact: Consider a conic bundle over a smooth incomplete curve $C$, i.e. a smooth surface $S$ with a proper surjective morphism to $C$ such that the pushforward of the structure sheaf of $S$ coincides with the structure sheaf of $C$, and the anticanonical class of $S$ is ample over $C$. If the base field is perfect, a conic bundle always extends to a conic bundle over a completion of $C$. I will tell about a necessary and sufficient condition for the existence of such an extension in the case of an arbitrary base field. The talk is based on a joint work in progress with V. Vologodsky.

## Speaker: Arnaud Beauville (University of Nice)

Title: Symmetric tensors on the intersection of two quadrics and Lagrangian fibration

Abstact: Let $X$ be a $n$-dimensional (smooth) intersection of two quadrics, and let $T^{*} X$ be its cotangent bundle. I will show that the algebra of symmetric tensors on $X$ is a polynomial algebra in $n$ variables. The corresponding $\operatorname{map} T^{*} X \rightarrow C^{n}$ is a Lagrangian fibration, which admits an explicit geometric description; its general fiber is an open subset of an abelian variety, again with a precise geometric description.
This is joint work with A. Etesse, A. Höring, J. Liu, C. Voisin.
Speaker: Thibaut Delcroix (University of Montpellier)
Title: Effective K-stability of spherical varieties
Abstact: The complexity of the action of a connected complex reductive group G on a (normal) algebraic variety $X$ is the minimal codimension of an orbit in $X$ of a Borel subgroup of $G$. As suggested by the name, low complexity group actions are easier to undrstand, the flagship example being toric varieties (that is, varieties with a complexity zero action of a torus $\left.G=(C *)^{n}\right)$. The class of varieties with a complexity zero group action is actually much larger, and also known as the class of spherical varieties.
K-stability, one of the main topic of the conference, is the algebraic counterpart to existence of canonical Kähler metrics underlying the Yau-TianDonaldson conjecture. I will explain how and when, for spherical varieties, K-stability conditions can effectively be computed. Applications to canonical Kähler metrics on projective and affine spherical varieties will also be discussed.

## Speaker: Young-Hoon Kiem (KIAS)

Title: Counting surfaces in projective varieties
Abstact: One of the oldest problems in algebraic geometry is to enumerate points, curves and surfaces in a given projective variety satisfying certain given
conditions. Point counting is handled by intersection theory. To count curves or surfaces, we construct compactified moduli spaces of curves or surfaces and apply intersection theory. As there are many ways to compactify, we have many ways to count them. In the first hour, I will review known methods to virtually enumerate curves such as Seiberg-Witten, Gromov-Witten and Donaldson-Thomas invariants. In the second hour, I will talk about recent advances in surface counting in Calabi-Yau varieties.

## Speaker: Fabrizio Catanese (University of Bayreuth)

Title: Status of the classification and old and new constructions for surfaces of general type with $p_{g}=q=2$

Abstact: The classification of surfaces of general type with $p_{g}=q=2$ is an intriguing and open chapter of surface theory.
For these surfaces, apart from the elementary cases where the Albanese image is a curve, respectively when $K^{2}$ attains its minimal value 4 , we have examples with $K^{2}=5,6,7,8$ and with degree $d$ of the Albanese map in the set $\{2,3,4,6\}$.
A component of their moduli space is said to be of the main stream if the map associating to a surface $S$ its Albanese surface $A=\operatorname{Alb}(S)$ has image of dimension 3 (hence the component dominates a component of the moduli space of Abelian surfaces).
I shall illustrate the status of the classification, and I will first show Penegini's examples of components not of the main stream, given by surfaces isogenous to a product (this is the only case where degree $d=6$ is attained).
I will then show very simple equations for some components of the main stream, named CHPP, PP4, AC3 surfaces, the letters standing for the names of several authors: Chen-Hacon, Penegini-Polizzi, Alessandro-Catanese (here $K^{2}=5,6,6, d=3,4,3$ ).
I shall then describe joint work with Edoardo Sernesi, concerning the branch curve of an Abelian surface with a polarization of type ( 1,3 ): this enables to show the existence of the family AC3.
I shall then describe how these components, in view of the Fourier Mukai transform, can be characterized via some assumption on the Albanese map.
Time permitting, I shall explain why the existence of such surfaces with $K^{2}=9$ is still open, in spite of some recent results.

