Workshop on Classical Algebraic Geometry

IBS Center for Complex Geometry

December 12-15, 2023

Speaker: **Alberto CALABRI**

Title: *On the lengths of plane Cremona transformations*

Abstact: A plane Cremona transformation is a birational map of the projective plane in itself and it is defined by three homogeneous polynomials, with no common factor, of the same degree. We will review the properties of the algebraic variety parametrizing plane Cremona transfomations of fixed degree. We will recall the definition by Blanc and Furter of the length of a plane Cremona map as the the minimal number of Jonquières maps needed in order to decompose it. We will introduce the notion of (ordinary) quadratic length of a plane Cremona map as the the minimal number of (ordinary) quadratic maps needed in order to decompose it. We will then report on a joint work in progress with Nguyen Thi Ngoc Giao about lower and upper bounds for the (ordinary) quadratic length of plane Cremona maps of small degree.

Speaker: **Cinzia CASAGRANDE**

Title: *Fano 4-folds with $b_2 > 12$ are products of surfaces*

Abstact: Let X be a smooth, complex Fano 4-fold, and b_2 its second Betti number. We will discuss the following result: if $b_2 > 12$, then X is a product of del Pezzo surfaces. The proof relies on a careful study of divisorial elementary contractions $f : X \to Y$ such that the image S of the exceptional divisor is a surface, together with my previous work on Fano 4-folds. In particular, given $f : X \to Y$ as above, under suitable assumptions we show that S is a smooth del Pezzo surface with $-K_S$ given by the restriction of $-K_Y$.

Speaker: **Flaminio FLAMINI**

Title: *Ulrich bundles on some 3-fold scrolls over Hirzebruch surfaces*

Abstact: In this talk I will report on a recent joint work (and also work in progress) with M. L. Fania (University of L'Aquila), where we investigate on Ulrich bundles on suitable 3-fold scrolls. More precisely inspired by previous results, concerning Ulrich bundles of low rank on scrolls over Hirzebruch

surfaces F_0 and F_1, which arise as 3-folds of low degree in projective spaces, we explicitely describe components $M(r)$ of moduli spaces of Ulich bundles of any rank $r > 0$ on 3-fold scrolls X_e over Hirzebruch surfaces F_e, for any non-negative integer e, showing that these components $M(r)$ are generically smooth, of computed dimension and whose general point is proved to correspond to a slope-stable bundle. These results allow us to further compute the Ulrich complexity of the 3-folds X_e and to prove also that they are varieties of Ulrich wild type. If time permits, some ideas addressing the birational geometry of such components $M(r)$ will be also discussed.

Speaker: **Paola FREDIANI**

Title: *On the local geometry of the moduli space of (2,2)-threefolds in A_9*

Abstact: We will present a study of the local geometry of the moduli space of intermediate Jacobians of (2,2)-threefolds in $\mathbb{P}^2 \times \mathbb{P}^2$. More precisely, we will prove that a composition of the second fundamental form of the Siegel metric in A_9 restricted to this moduli space, with a natural multiplication map, is a nonzero holomorphic section of a vector bundle. We will also give an explicit description of its kernel. This is a joint work with Elisabetta Colombo, Juan Carlos Naranjo and Gian Pietro Pirola.

Speaker: **Kangjin HAN**

Title: *On defining equations of some k-secants of Veronese varieties*

Abstact: In this talk, we report some results on the equations of $\sigma_k(v_d(\mathbb{P}^n))$, k-th secant variety of d-uple Veronese embedding of a projective space, for some small k, d, n. Knowledge of defining equations of higher secants is classically important in itself and in recent years it also has drawn strong attention in symmetric tensor rank problems. We first recall known results on the equation of secants and related notions of 'k-secant variety of minimal degree' and 'del Pezzo k-secant variety', which is due to Ciliberto-Russo and Choe-Kwak, respectively. Next, we explain our method and its consequences. This is a joint work with K. Furukawa.

Speaker: **Zhi JIANG**

Title: *On surfaces of general type with $p_g = q = 2$*

Abstact: We will report some recent progress on the classification of minimal surfaces of general type with $p_g = q = 2$.The two main ingredients of our approach are the Chen-Jiang decomposition and the cohomological rank functions of the Albanese-pushforward of the canonical bundle. This talk is based on a joint work with Jiabing Du and Guoyun Zhang.

Speaker: **Akihiro KANEMITSU**

Title: *$K3$ surfaces of genus 13 and curves of genus 3*

Abstact: Let C be a hyperelliptic curve of genus 3 and L a line bundle of odd degree. The moduli space N of rank 2 stable bundles with determinant L is embedded into the Grassmann variety Gr(8, 2) as the zero locus Gr(8, 2, P) of a pencil P of quadric forms (Desale-Ramanan).

We give a description of $K3$ surfaces of genus 13 in N by using vector bundles on N. And we discuss some generalizations of this description. This is a joint work with Professor Shigeru Mukai.

Speaker: **Grzegorz KAPUSTKA**

Title: *Constructions of derived equivalent hyper-Kahler fourfolds*

Abstact: We present conditions under which pairs of hyper-Kahler fourfolds of $K3^{[2]}$-type of Picard rank 1 are derived equivalent. Moreover we describe new constructions of pairs of twisted derived equivalent hyper-Kahler manifolds manifolds of Picard rank > 1. This is a joint work in progress with Michal Kapustka.

Speaker: **Michał KAPUSTKA**

Title: *Nodal surfaces and hyperkaehler manifolds*

Abstact: Surfaces with even sets of nodes are nodal surfaces which admit smooth double covers branched along their nodes. These are classical objects appearing in various contexts in algebraic geometry. In this talk, we will discuss several constructions of surfaces with even sets of nodes which are naturally related to some hyperkaehler manifolds. We shall link these surfaces to Fano manifolds of $K3$ type or $K3$ surfaces by means of conic fibrations and Hodge structures. These surfaces appeared in two different projects involving two groups of people and their study is now an ongoing project joint with: M. Bernardara, E. Fatighenti, G. Kapustka, L. Manivel, G. Mongardi, and F. Tanturri.

Speaker: **Seonja KIM**

Title: *Family of smooth curves on cones and their Tschirnhausen modules*

Abstact: Let $I(d, g, r)$ be the Hilbert scheme whose general points represent smooth irreducible non-degenerate curves of degree d and genus g in $\mathbb{P}^r$. In 1921 Severi claimed that $I(d, g, r)$ is irreducible when $d \geq g + r$. This was proved in case $r \leq 4$ but several counterexamples have appeared for $r \geq 6$. Along this line we will discuss about the geometry of $I(d, g, r)$ by using smooth curves on cones and their Tschirnhausen modules.

Speaker: **Hirokazu NASU**

Title: *Obstructions to deforming curves on a Fano threefold and non-reduced components of Hilbert schemes*

Abstact: Computing obstructions is useful for determining the dimension and the singularity of a Hilbert scheme at a given point, but it is a hard task if the obstruction space is nonzero. In a previous joint research with S. Mukai, we have developed a technique to compute obstructions to deforming curves on a (uni-ruled) threefold. In the computations, we have made use of an intermediate smooth surface and also some special (rational or elliptic) curves on the surface. Following this research, I have recently developed the technique further and have obtained many examples of non-reduced components of Hilbert schemes. The research on finding such components was started by Mumford (1962) in the case of the Hilbert scheme of space curves (as a pathology in AG). In this talk, I will talk about its generalization to the Hilbert scheme of smooth Fano threefold of Picard rank one. If the time allows, I will also report on a recent progress on a conjecture due to J. O. Kleppe and Ph. Ellia, which is concerned with the dimension of the Hilbert scheme of space curves (lying on a smooth cubic surface).

Speaker: **Wenbo NIU**

Title: *Fundamental forms of algebraic varieties*

Abstact: Fundamental forms can be thought of as linear systems attached to the projectivization of Zariski tangent space at a nonsingular point of a variety. It was developed by the method of moving frames in differential geometry. In 1979, Griffiths-Harris used fundamental forms to study geometry of algebraic varieties and observed some vanishing phenomena. In this talk, I discuss an algebraic approach to the theory of fundamental forms using sheaves of principal parts instead of moving frames. Furthermore, I will extend the vanishing of the third fundamental forms obtained by Griffiths-Harris and Landsberg to the fundamental forms of arbitrary order. This is a joint work with L. Ein.

Speaker: **Jinhyung PARK**

Title: *Syzygies of secant varieties of smooth projective curves*

Abstact: In this talk, I report recent progress on syzygies of secant varieties of smooth projective curves. First, we extend Green's $(2g+1+p)$-theorem to secant varieties of smooth projective curves. This confirms Sidman-Vermeire's conjecture. This part is joint work with Lawrence Ein and Wenbo Niu. Next, we show a generalization of the gonality conjecture on syzygies of smooth projective curves to their secant varieties. More precisely, we prove that the gonality sequence of a smooth projective curve completely determines the shape of the minimal free resolutions of secant varieties of curves of large degree. This answers a question of Ein. This part is joint work with Junho Choe and Sijong Kwak. Our results show that there is a "matryoshka structure" among secant varieties of smooth projective curves.

Speaker: **Xavier ROULLEAU**

Title: *Regular polygons, line operators, and elliptic modular surfaces*

Abstact: Joint work with Lukas Kühne. Line arrangements have been used in various domains of mathematics, such as Topology, Combinatorics, Algebra... An important example in Algebraic Geometry is the first geometric construction of ball quotient surfaces, done in the eighties by Hirzebruch, by using covers of the plane branched over some special line arrangements.

In the search for new interesting line arrangements, I proposed to define some geometric operators transforming a line arrangement into another one. For example, one may define $\Lambda_{\{2\},\{3\}}$ as the map which to a line arrangement L returns the union of the lines containing exactly three points among the double points of L.

It is then a natural question to construct moduli spaces of line arrangements which are preserved by these operators, so that one obtains a natural dynamical system, and one may expect interesting features of the periodic elements.

In this talk I will explain how we constructed, by using the regular n-gons, infinitely many such moduli spaces of line arrangements preserved by some geometric operators. It turns out that these moduli spaces are also the modular elliptic surfaces over the modular curves $X_1(n)$, and the operators acts on these surfaces as the multiplication by -2 map. We obtain in that way a geometric construction of the elliptic modular surfaces first constructed by Shioda, by using complex uniformization. If time permits, I will also explain how to construct the modular curves $X_1(N)$ as moduli spaces of points arrangements.

Speaker: **Francesco RUSSO**

Title: *On rational complete intersections of three quadrics in* $\mathbb{P}^7$

Abstact: A very general smooth complete intersection of three quadrics in $\mathbb{P}^7$ is not stably rational and there exist infinitely many irreducible loci in the corresponding Hilbert scheme which parametrize rational objects and whose union is dense in the euclidean topology (Hassett, Pirutka and Tschinkel, 2017).

We shall briefly revisit and reformulate this construction to produce explicitly some of these loci and to show the difficulties of the effective construction of a birational map to $\mathbb{P}^4$ for a general element in the corresponding families.

Then we shall describe explicit birational maps from some of these rational complete intersection of three quadrics in $\mathbb{P}^7$ to some prime Fano manifolds together with their Sarkisov decomposition via a single *Secant Flop*, allowing us to recover the *cohomologically* associated Castelnuovo surface of general type with $K^2 = 2$ and $\chi = 4$ (the double cover of $\mathbb{P}^2$

ramified along the discriminant curve of the net of quadrics defining the complete intersection) as the minimal model of the non ruled irreducible component of the base locus of the inverse maps. These examples are the analogous to those produced for cubic or for Gushel-Mukai fourfolds and they provide a geometric realization of the abstract Hodge theoretic O' Grady isomorphism between primitive 2-cycles on the complete intersection and primitive 1-cycles on the associated surface.

This is joint work with Giovanni Staglianò.

Speaker: **Eduardo Luis SOLÁ CONDE**

Title: *Birational transformations associated to torus actions and applications*

Abstact: In this seminar I will present some results belonging to a project in collaboration with G. Occhetta, E. Romano, J. Wisniewski, et al. We will discuss actions of the 1-dimensional complex torus on projective varieties, and study their relation with certain types of birational transformations. We will also present some examples of this relation in the framework of rational homogeneous varieties, and explain briefly how these tools can be used to characterize rational homogeneous spaces.