School on Moduli, K-trivial Varieties, and Related Topics

IBS Center for Complex Geometry

February 21-23, 2024

Speaker: **Harold Blum**

Title: *Moduli of Fano varieties and K-stability*

Abstact: K-stability is an algebraic notion that was introduced by differential geometers to characterize when a Fano variety admits a Kahler-Einstein metric. While its origins are analytic, recently it has been applied to construct an algebraic moduli theory for Fano varieties. In this lecture series, I will first explain the motivation for K-stability and how to understand the definition using various tools from algebraic geometry. Then I will discuss how and why K-stability is used to construct a moduli theory for Fano varieties. Finally, I will describe some explicit examples of these moduli spaces and how they are computed.

Speaker: **Radu Laza**

Title: *Introduction to the moduli of K-trivial varieties*

Abstact: The K-trivial varieties are one of the fundamental building blocks in Algebraic Geometry. They come into three flavors: complex tori/abelian varieties, hyper-Kaehler manifolds, and (strict) Calabi-Yau manifolds. In these lectures, I will review the moduli theory of those, both the local and global picture. The basic thrust of the lectures is that Hodge theory controls the moduli space of K-trivial varieties. In the classical case, that of abelian varieties and hyper-Kaehler manifolds, one has a fairly good understanding of the global geometry of the moduli space. This is due to the fact that the period domain, parametrizing the relevant Hodge structures, is of Hermitian symmetric type. In contrast, in the case of Calabi-Yau threefolds (and higher dimensions), the picture is more subtle, and one has only a partial understanding. I will discuss some new results regarding the local moduli (=deformation theory) of singular Calabi-Yau varieties of higher dimensions, and I will speculate on a potential global picture, that mirrors the situation from the classical case.

Speaker: **Colleen Robles**

Title: *Hodge structures, period maps and degenerations*

Abstact: The goal of these three lectures is to introduce Schmid's nilpotent orbit theorem, and to explain its role in constructing completions of period maps. We will cover (mixed) Hodge structures, period maps (variations of Hodge structure), and their degenerations. If time allows we will relate the nilpotent orbits of Schmid's theorem to the Satake-Baily-Borel compactification of a locally hermitian symmetric space, and its toroidal normalization by Ash-Mumford-Rapoport-Tai.