

Workshop on Moduli, K -trivial Varieties, and Related Topics

IBS Center for Complex Geometry

February 21-23 and 26-29, 2024

Speaker: **Kenneth Ascher**

Title: *Wall crossing for moduli spaces of higher dimensional varieties*

Abstract: Wall crossing, which originated in Hassett's work on stable pointed curves, has emerged as a powerful technique for studying compactifications of higher dimensional varieties. I will review several results in this setting, and discuss applications of this theory to some concrete moduli problems.

Speaker: **Philip Engel**

Title: *Compact moduli of Enriques surfaces*

Abstract: The moduli space of Enriques surfaces is an orthogonal Shimura variety of dimension 10. In this capacity, it has a natural infinite class of so-called "toroidal" compactifications. Is any one of these toroidal compactifications distinguished, in the sense that it parameterizes some generalized or "stable" Enriques surfaces? I will describe joint work with V. Alexeev, D. Garza, and L. Schaffler giving an affirmative answer to this question, for Horikawa's model of Enriques surfaces.

Speaker: **Laure Flapan**

Title: *Modular Forms and Divisors on Orthogonal Modular Varieties*

Abstract: We describe how the vanishing of modular forms along certain divisors can be used to prove general-type results for orthogonal modular varieties, in particular new general-type results for moduli spaces of polarized Hyperkähler manifolds of $K3^{[n]}$ type. These techniques also yield an approach to providing a negative answer to the question of whether the effective cone of such an orthogonal modular variety, particularly in the case of moduli spaces of quasi-polarized K3 surfaces, is generated by irreducible components of Noether-Lefschetz divisors. This is joint work with Ignacio Barros, Pietro Beri, and Emma Brakkee.

Speaker: **Yoon-Joo Kim**

Title: *Isotrivial fibrations of hyper-Kähler manifolds*

Abstract: I would like to report recent joint work with R. Laza and O. Martin about isotrivial fibrations of compact hyper-Kähler (HK) manifolds. A Lagrangian fibration $f : X \rightarrow B$ of a HK $2n$ -fold is called isotrivial if its smooth fibers are all isomorphic to a single n -dimensional abelian variety F . Given an isotrivial fibration $f : X \rightarrow B$ to a smooth base B with a rational section, we show the following four statements: (1) F is isogenous to the n -th power of an elliptic curve E . (2) There are two types of isotrivial fibrations (called type A and B) with different behaviors. (3) If the elliptic curve factor E of F has no complex multiplication by $\sqrt{-1}$ or $\sqrt{-3}$, then f must be of type A. (4) If f is of type A, then X is birational to the Hilbert scheme of a K3 surface or a generalized Kummer variety. Our current proof of (4) depends on the smoothness conjecture on the Lagrangian base of HK manifolds.

Speaker: **Kyoung-Seog Lee**

Title: *Symmetric products, Jacobians and moduli spaces of vector bundles of algebraic curves*

Abstract: Symmetric products, Jacobians and moduli spaces of vector bundles on curves are fundamental objects in the study of algebraic curves. In this talk, I will explain how they are related in the level of their derived categories and motives. This talk is based on several joint works with I. Biswas, T. Gomez, H.-B. Moon and M. S. Narasimhan.

Speaker: **Zhiyuan Li**

Title: *Zero cycles on Bridgeland moduli spaces*

Abstract: In this talk, I will talk about some recent progress on the study of zero cycles on moduli space of Bridgeland stable objects in some K3 category. The motivation is so called the Beauville-Voisin conjecture which predicts the existence of a filtration on the Chow groups of projective hyper-Kähler manifolds opposite to the conjectural Bloch-Beilinson filtration. I will discuss the construction of such filtration on zero cycles and explain its applications to the Bloch's conjecture for birational automorphisms of hyper-Kähler varieties. This is due to the joint work with X. Yu, H. Guo, R. Zhang and X. Zhang.

Speaker: **Yuchen Liu**

Title: *Hassett-Keel program in genus four*

Abstract: The Hassett-Keel program aims to find modular meanings for the minimal model program of the Deligne-Mumford compactification of moduli space of curves. The first three steps of the Hassett-Keel program have been

established in all genera thanks to the work of Hassett, Hyeon, Alper, Fedorchuk, and Smyth. In genus four, the last several steps of the program have been worked out by Casalaina-Martin, Laza and Jensen using variation of GIT. In this talk, I will explain how to fill in the gap between these two approaches and thus to complete the Hassett-Keel program in genus four. Our method involves the recently developed moduli theory of log Calabi-Yau pairs, as well as explicit wall crossing to replace hyperelliptic curves and ribbons. Based on joint work in progress with Kenneth Ascher, Kristin DeVleming, and Xiaowei Wang.

Speaker: **Eyal Markman**

Title: *Cycles on abelian 2n-folds of Weil type from secant sheaves on abelian n-folds*

Abstract: Let X be an abelian n -fold, $n \geq 2$, and \hat{X} its dual abelian n -fold. The even cohomology $H^{\text{even}}(X, \mathbb{Z})$ is the half spin representation of an arithmetic subgroup of $\text{Spin}(4n)$. Fix an integer $d > 0$ and set $K := \mathbb{Q}[\sqrt{-d}]$. A coherent sheaf F on X is a K -secant sheaf, if $ch(F)$ belongs to a 2-dimensional subspace P of $H^{\text{even}}(X, \mathbb{Q})$ spanned by Hodge classes, such that the line $\mathbb{P} := \mathbb{P}(P)$ intersects the spinorial variety in $\mathbb{P}[H^{\text{even}}(X, K)]$ along two distinct complex conjugate points. The K -secant \mathbb{P} determines an embedding $\eta : K \rightarrow \text{End}_{\mathbb{Q}}(X \times \hat{X})$ and a non-degenerate 2-form h on $X \times \hat{X}$. The triple $(X \times \hat{X}, \eta, h)$ is a polarized abelian variety of Weil type, for a non-empty open subset of such K -secants.

Let F_1, F_2 be coherent sheaves on X with $ch(F_i)$ in such a K -secant \mathbb{P} . Assume that Orlov's equivalence $\Phi : D^b(X \times X) \rightarrow D^b(X \times \hat{X})$ maps $F_1^\vee \boxtimes F_2$ to a shift of a vector bundle E . We prove that the characteristic class $\exp\left(-\frac{c_1(E)}{\text{rank}(E)}\right) ch(E)$ of the projective bundle $\mathbb{P}(E)$ remains of Hodge type under every deformation of $(X \times \hat{X}, \eta, h)$ as a polarized abelian variety of Weil type. The algebraicity of the Hodge-Weil classes of a deformation of the triple would follow, if $\mathbb{P}(E)$ deforms as well.

When $n = 3$ we construct for every positive integer d an example, where E is a rank $8d$ simple vector bundle over $X \times \hat{X}$. We show (work in progress) that $\mathbb{P}(E)$ deforms with $(X \times \hat{X}, \eta, h)$ locally in the 9-dimensional moduli space of polarized abelian 6-folds of Weil type with complex multiplication by $\mathbb{Q}[\sqrt{-d}]$, conditional on a conjecture that an unobstructedness theorem of Buchweitz-Flenner for deformations of semiregular coherent sheaves generalizes to semiregular projective bundles.

Speaker: **Mirko Mauri**

Title: *An algebro-geometric version of the Poincaré conjecture*

Abstract: Dual complexes are CW-complexes, encoding the combinatorial data of how the irreducible components of a simple normal crossing pair intersect.

The algebraic geometry of the divisor is reflected in the topology of the dual complex. One of the most tantalizing conjectures in the field is the expectation that the dual complex of an anticanonical divisor is a sphere or a finite quotient of a sphere. Equivalently, a combinatorial Calabi-Yau variety should resemble a sphere.

I will provide an overview of the current state of the conjecture, and report on joint work with Joaquin Moraga. We introduce a numerical invariant called birational complexity that, among other properties, measures the degree to which the dual complex of an anticanonical divisor is close to be a sphere.

Speaker: **Enrica Mazzon**

Title: *A non-archimedean approach to the SYZ conjecture*

Abstract: The SYZ conjecture concerns degenerations of complex Calabi-Yau manifolds and was proposed as a geometric explanation of mirror symmetry. Kontsevich and Soibelman introduced a non-archimedean approach to this conjecture, and more recently, Yang Li's work has connected the non-archimedean approach with the original SYZ conjecture. In this talk, I will explain the key concepts of the non-archimedean approach and present recent developments in the context of hypersurfaces. This is based on a project in collaboration with Jakob Hultgren, Mattias Jonsson and Nick McCleerey.

Speaker: **Yuji Odaka**

Title: *Moduli compactifications of polarized K -trivial varieties and connections with Ricci-flat Kahler metrics*

Abstract: Each smooth K -trivial projective variety admits Ricci-flat Kahler metrics (a.k.a. Calabi-Yau metrics or a case of Kahler-Einstein metrics), for each ample line bundle, or Kahler class in general by the theorem of Yau. On the other hand, moduli or deformation of K -trivial varieties are classical topics in algebraic geometry. I plan to make some survey talk about how these can interact closely, especially with focus on several of my works (which may well include some joint works as I will clarify in the talk).

Speaker: **Kieran O'Grady**

Title: *Moduli spaces of semistable sheaves on HK fourfolds*

Abstract: I will report on work in progress whose goal is to prove that on a general polarized HK variety of type $K3^{[2]}$, the moduli space of semistable sheaves with suitable ranks and first two Chern classes contains a connected component which is a general polarized HK variety of type $K3^{[n]}$ (for $n = a^2 + 1$, $a = 1, 2, 3$, etc).

Speaker: **Keiji Oguiso**

Title: *Fibered Calabi-Yau threefolds with relative automorphisms of positive entropy and c_2 -contractions*

Abstract: I would like to discuss the uniqueness of an abelian fibered Calabi-Yau threefold with an automorphism of positive entropy, preserving the fibration, together with non-trivial relevant explicit examples. I would also like to give a fairly concrete structure theorem of an elliptically fibered Calabi-Yau threefold with an automorphism of positive entropy, preserving the fibration. The second Chern class plays a key role in our results.

Speaker: **Gianluca Pacienza**

Title: *Regenerations and applications*

Abstract: Chen-Gounelas-Liedtke recently introduced a powerful regeneration technique, a process opposite to specialization, to prove existence results for rational curves on projective K3 surfaces. In the talk I will present a joint work with G. Mongardi in which we show that, for projective irreducible holomorphic symplectic manifolds, an analogous regeneration principle holds and provides a very flexible tool to prove existence of uniruled divisors, significantly improving known results.

Speaker: **Colleen Robles**

Title: *Completions of 2-parameter period maps by nilpotent orbits*

Abstract: The Satake-Baily-Borel compactification (SBB) of a locally hermitian symmetric space, and its toroidal desingularizations by Ash-Mumford-Rapoport-Tai (AMRT), yield Hodge-theoretic completions of certain moduli spaces (including abelian varieties). It is a long-standing problem, with origins in questions and conjectures posed by P. Griffiths in 1970, to develop analogs of these compactifications when the period domain is not hermitian. I will report on work that establishes the desired analogs for 2-parameter period maps. The SBB-analog is joint with M. Green, P. Griffiths and R. Laza. The AMRT-analog is based on the proposed Kato-Usui construction, and is joint work with H. Deng.

Speaker: **Justin Sawon**

Title: *Twists of Lagrangian fibrations*

Abstract: We consider (Tate-Shafarevich) twists of Lagrangian fibrations $X \rightarrow B$ on holomorphic symplectic manifolds. These are fibrations $X' \rightarrow B$ that locally over the base B are isomorphic to $X \rightarrow B$. We describe the cohomology groups parametrizing such twists and exhibit some interesting examples. Our main new discovery is that these twists can produce non-Kähler spaces from projective spaces; namely, twists of generalized Kummer varieties yield Guan's examples of non-Kähler holomorphic symplectic manifolds.

Speaker: **Qizheng Yin**

Title: *Generalized Beauville decompositions*

Abstract: The cohomology/Chow ring of an abelian scheme admits the Beauville decomposition, which is a Fourier-stable, multiplicative splitting of the Leray filtration. It is natural to ask whether such a decomposition exists for abelian fibrations with singular fibers. For compactified Jacobian fibrations, a weaker form of the Beauville decomposition was shown to hold in earlier joint work with Davesh Maulik and Junliang Shen. It explained conceptually why the perverse filtration for such fibrations is multiplicative, and eventually led to a proof of the $P = W$ conjecture. On the other hand, the (strong) Beauville decomposition does not hold for arbitrary abelian fibrations (not even for compactified Jacobian fibrations). In case of Lagrangian fibrations, the existence of a Chow-theoretic Beauville decomposition is closely related to the Beauville-Voisin conjectures for hyper-Kähler varieties. Based on joint work in progress with Younghun Bae, Davesh Maulik, and Junliang Shen.