Workshop on Geometry of Homogeneous Varieties

IBS Center for Complex Geometry

April 15-18, 2024

Speaker: **Michel Brion**

Title: *Minimal rational curves on almost homogeneous varieties*

Abstact: Families of minimal rational curves and the associated varieties of minimal rational tangents (VMRTs) feature prominently in the geometry of uniruled projective varieties. Almost homogeneous varieties form a remarkable class of such varieties, for which the group action helps determine the VMRTs. The talks will illustrate this for certain subclasses, including the complete symmetric varieties (based on joint work with Nicolas Perrin and Shinyoung Kim).

Speaker: **Minseong Kwon**

Title: *Spherical Geometry of Hilbert Schemes of Conics in Adjoint Varieties*

Abstact: In a given rational homogeneous variety, geometry of lines, or more generally linear subspaces, is well-understood. The next simplest non-linear subvarieties are conics, and in this talk, I will discuss conics in the adjoint varieties, which are rational homogeneous varieties defined as the projectivized minimal nilpotent orbits in the simple Lie algebras. First, I will show that the Hilbert schemes of conics in the adjoint varieties are complex-symmetric varieties, using the contact structure on the adjoint varieties. Next, I will explain how the colored fan of the Hilbert scheme of conics can be computed based on geometry of linear subspaces. As a corollary, a description of the conjugacy classes of conics in the adjoint varieties will be presented.

Speaker: **Qifeng Li**

Title: *Rigidity of rational homogeneous spaces of higher Picard numbers*

Abstact: In this talk, we will discuss the rigidity of the rational homogeneous space S. Two types of rigidity are considered, namely the rigidity under Fano deformation of S and the Schur rigidity of S as a homogeneous submanifold on another rational homogeneous space. When S is of Picard number one these rigidity problems are well studied by Hwang, Mok, Hong etc.

Our interest lies in the cases when S has a higher Picard number. To study these cases, one need to combine the methods in Picard number, such as Cartan geometry and the VMRT theory, with the theory of Mori fibrations.

Speaker: **Boris Pasquier**

Title: *Horospherical two-orbit varieties as zero loci (work with Laurent Manivel)*

Abstact: The classification of smooth horospherical varieties with Picard number one gives five families of two-orbit varieties. One of them is the family of odd symplectic grassmannians, which can be described as the zero locus of a general section of a homogeneous vector bundle over some grassmannian. We do not have the same result for the other families, but we could have an analogue by blowing-up the closed orbit.

In a first part, I will introduce these varieties and the way to construct them as zero loci.

In a second part, I will discuss, as an application, a way to compute the cohomology ring of these varieties.

Speaker: **Kyeong-Dong Park**

Title: *Equivariant Ulrich bundles on rational homogeneous varieties of Picard number one*

Abstact: An Ulrich bundles on a projective variety is an initialized aCM (arithmetically Cohen-Macaulay) bundle having the maximum possible number of global sections. Ulrich bundles are vector bundles which enjoy many special features, and the existence and properties of Ulrich bundles on a given algebraic variety tell us geometric properties of the variety. Eisenbud and Schreyer asked whether every projective variety admits an Ulrich bundle and their question has been answered positively for several cases. In particular, Costa and Miro-Roig classified irreducible equivariant Ulrich bundles on Grassmannians. However, unlike Grassmannians of type A, only some special rational homogeneous varieties of other Lie types admit irreducible equivariant Ulrich bundles. As a result, in order to find Ulrich bundles on homogeneous varieties, we need to consider reducible equivariant vector bundles on them. When a rational homogeneous varieties of Picard number one is described as the zero locus for a general global section of certain equivariant vector bundle on a Grassmannian, a specific construction method for equivariant Ulrich bundles will be explicitly discussed. This talk is based on joint works with Kyoung-Seog Lee.

Speaker: **Lea Villeneuve**

Title: *Geometry of horospherical varieties of rank 1, smooth, projective, with Picard number 2*

Abstact: Horospherical varieties represent an important subfamily of complex algebraic varieties, endowed with the action of a reductive and connected group G. The ones which are smooth with Picard number 1 (non homogeneous) are grouped into 5 families and have already been studied. The ones which are smooth with Picard number 2 are much more numerous. In this talk, I will study the geometry of these varieties X, only in rank 1, which already represents more than 500 families. Part of the geometry of theses varieties has already been described by the Log MMP, but I will give other types of geometric characteristics here. The first part of the study will deal with the action of the connected automorphisms group of X, which has 3 orbits under the action of G. In particular, I will show that X is either a two or three-orbit variety under the action of the connected automorphisms group. Then, a full description of this group will be given and I will describe the automorphisms group of X. The second part of the talk will deal with the local rigidity of X.

Speaker: **Yoshinori Namikawa**

Title: *Toric hyperkahler varieties and related topics*

Abstact: A toric hyperkahler variety is defined as a hyperkahler reduction of a quaternionic space with the standard hyperkahler structure by an action of a compact torus. If we fix a complex structure of the quaternionic space, it can be regarded as a holomorphic symplectic reduction of a complex affine space by an action of an algebraic torus. It provides us with an example of a conical symplectic singularity. In this lecture, we deal mainly with two topics:

(1) a toric hyperkahler construction of isolated symplectic singularities with trivial local fundamental group, and

(2) a $\mathbb{Q}$-factorial terminalization of an affine toric hyperkahler variety and an explicit description of the universal Poisson deformation of the affine toric hyperkahler variety.

Speaker: **Thibaut Delcroix**

Title: *Bubbles on (horo)-symmetric varieties*

Abstact: I will present how Stenzel's Kähler Ricci-flat metrics appear as bubbles in the degeneration of conical Kähler-Einstein metrics on rank 1 horosymmetric varieties (compactifications of homogeneous fibrations with rank 1 complex symmetric spaces fibers). This family of examples contains the case (highlighted numerically by Chi Li) of the Eguchi-Hanson metric as a bubble in the convergence of Kähler-Einstein metrics on $\mathbb{P}^2$, with conic singularities along a quadric, to an orbifold Kähler-Einstein metric on $\mathbb{P}(1, 1, 4)$. Time permitting, I will then discuss possible higher-rank extensions, where the asymptotically conical Calabi-Yau metrics on symmetric spaces constructed by Biquard and Gauduchon, Biquard and myself, and by my student Nghiem, should play the role of bubbles.

Speaker: **Jarek Buczynski**

Title of Talk 1: *Algebraic torus actions on smooth projective manifolds*

Title of Talk 2: *Contact Fano manifolds*

Abstact: The joint goal of the series of two talks is to present a classification result about contact Fano manifolds in low dimensions, however the first part might be of independent and more general interest.

A complex manifold is a contact manifold if there is a distribution in the tangent bundle which is as non-integrable as possible. I will report on recent progress in the classification of projective contact manifolds focusing on the Fano case. A conjecture of LeBrun and Salamon asserts that all such are homogeneous spaces. We prove the conjecture in (complex) dimension 7 and 9. Our work implies also the classification of positive quaternion-Kaehler manifolds of (real) dimensions 12 and 16, solving a famous problem from Riemannian geometry. The tools we use include representation theory and actions of (complex) reductive groups on manifolds, symplectic geometry, characteristic classes, and equivariant localisation theorems. In particular, during the first talk I will focus on more general problems: 1) how to recognise a homogeneous space in a relatively abstract projective manifold X, 2) describing an action of an algebraic torus on a projective manifold. During the second talk I will explain the applications of these methods to the LeBrun-Salamon conjecture.