Workshop on Algebraic Geometry in Busan

May 14-16, 2024

Speaker: **Lorenzo Barban**

Title: *Geometric realizations of small modifications of dream type*

Abstact: In this talk we establish a correspondence between algebraic torus actions and birational geometry. Given a birational map ϕ among normal projective varieties, we introduce the notion of *geometric realization of ϕ*, i.e. a normal projective $\mathbb{C}^*$-variety such that the birational map among geometric quotients parametrizing general orbits coincides with ϕ. Geometric realizations can be thought of as a projective algebraic version of the notion of cobordism coming from Morse theory. We show how to construct geometric realizations of maps which are *small modifications of dream type*, that is birational maps which are isomorphism in codimension 1 associated to a finitely generated multisection ring. We conclude by giving some insight of how to construct explicit geometric realizations for birational maps among toric varieties. This talk is based on joint works with E. Romano, G. Occhetta, L. Solá Conde and S. Urbinati.

Speaker: **Jungkai Chen**

Title: *Irregular Varieties with Small Invariants*

Abstact: In birational classification theory, irregular varieties are considered to be more well-behaved. Recently, it is proved by Hu and Zhang that for irregular threefolds of general type, one has $\mathrm{vol}(X) \geq \frac{4}{3}\chi(\omega_X)$, which is called the Noether-Severi Inequality. Moreover, when equality holds, one can characterize those varieties explicitly. In this talk, we will work on irregular varieties which are close to the Noether-Severi line. We will also investigate the lower bound of volume of irregular threefolds and the birationality problem. This is a joint work in progress with Zhi Jiang.

Speaker: **Toshiyuki Katsura**

Title: *Kummer surfaces and quadratic line complexes*

Abstract: The lines in the 3-dimensional projective space $\mathbf{P}^3$ are parametrized by a nonsingular quadric surface G (a Grassmann manifold) in $\mathbf{P}^5$. In the late

19th century, Klein took one more quadric Q in $\mathbf{P}^5$ and considered the pencil made by G and Q over the field of complex numbers . By using the pencil, he found a beautiful geometry which includes the lines in $\mathbf{P}^3$, curves C of genus 2, their Jacobian varieties $J(C)$ and Kummer surfaces $\mathrm{Km}(J(C))$. His theory holds also over the field of positive characteristic $p \neq 2$. However, if $p = 2$, the situation is very different and special phenomena appear. In this talk, I will first give a survey of the classical theory by Klein and then explain the theory which we established in the case of characteristic 2. This is a joint work with S. Kondo.

Speaker: **Shigeyuki Kondo**

Title: *Kummer quartic surfaces in characteristic two: bitangent lines and the automorphism group*

Abstact: The Kummer quartic surface was first found by a physicist A-J. Fresnel in 1822 in special case, and by E. Kummer in 1864 in general case. In case the characteristic is different from two, Kummer quartic surfaces have sixteen nodes, on the other hand, in characteristic two, the situation is completely different. In this talk, I would like to discuss the bitangent lines and the automorphism group of the Kummer quartic surface in characteristic two.

Speaker: **Ching-Jui Lai**

Title: *On movable cones of some Calabi-Yau threefolds of Picard rank two*

Abstact: The birational geometry of Calabi-Yau threefolds is intricate even with many advances in modern higher dimension geometry. For example, Reid's fantasy and the Morrison-Kawamata cone conjecture are only partially solved. In this talk, we report a recent joint work with Atsushi Ito (Okayama) and Sz-Sheng Wang (Academic Sinica at Taiwan), in which we describe explicitly the movable cones of a class of Calabi-Yau threefolds of Picard rank two. These examples may shed some light on the above-mentioned conjectures.

Speaker: **Donggun Lee**

Title: *Automorphisms and deformations of Hessenberg varieties*

Abstact: Hessenberg varieties are subvarieties in flag varieties which have interesting nontrivial symmetric group actions on their cohomology. The positivity of induced representations in the permutation module basis expansion is known to be equivalent to a long-standing conjecture proposed by Stanley-Stembridge in combinatorics.

To enhance our understanding of these representations, one might hope to identify a lift of the action on the cohomology to Hessenberg varieties themselves or to discover useful deformations of them. In this talk, we discuss automorphisms and deformations of Hessenberg varieties when

they are hypersurfaces in flag varieties. Especially in type A, we provide a complete classification along with an interpretation in terms of moduli of pointed rational curves. This is a joint work in progress with P. Brosnan, L. Escobar, J. Hong, E. Lee, A. Mellit and E. Sommers.

Speaker: **Hsueh-Yung Lin**

Title: *The effective cone conjecture*

Abstact: Let (X, D) be a klt Calabi-Yau pair. We formulate a version of the cone conjecture of (X, D) for the effective cone $\text{Eff}(X)$, and show that it is equivalent to the movable cone conjecture of (X, D) if we assume the existence of good minimal models. As an application, we prove (unconditionally) the movable cone conjecture for Schoen's Calabi-Yau threefolds. (Joint work in progress with C. Gachet, I. Stenger, and L. Wang.)

Speaker: **Shigeru Mukai**

Title: *Clifford's circle theorem and inseparable Kummer surfaces*

Abstact: Starting with $n + 1$ circles passing through a point, one obtains a configuration of 2^n points and 2^n circles. In the case $n + 1 = 6$, taking quotient by antipodal involution, this becomes Kummer's configuration of 16 nodes and 16 tropes. I will explain that this folded Clifford's configuration is realized in a 3-dimensional family of supersingular K3 surfaces in characteristic 2, as double plane (or double quadric). This family is a generalization of 2-dimensional one constructed by Kondo-Schroer(2021), and is studied recently by Matsumoto and Dolgachev. I will also explain a relation with Shimada's work (2004, 2006) on inseparable double plane K3 surfaces.

Speaker: **Keiji Oguiso**

Title: *Endomorphisms of a variety of Ueno type and some remarks*

Abstact: A variety of Ueno type is defined by the standard resolution of the quotient of the self-product of an elliptic curve by scalar multiplication. Varieties of Uneno type are quite simple but play several important and interesting roles in birational geometry and algebraic dynamics. After a brief review of some of such roles, I would like to show that the monoid of endomorphisms of self separable surjective morphism of a variety of Ueno type coincides with the group of automorphisms and give then the explicit description of the automorphism group. We then give a few applications together with some relevant applications to Calabi-Yau threefolds.