Workshop on Hyperkähler Varieties and Related Topics

March 10–13, 2025

Ekaterina Amerik

Title (Lecture I–III): *Parabolic automorphisms of hyperkähler manifolds*

Abstract: Let X be an irreducible holomorphic symplectic ("hyperkähler") manifold. Then $H^{1,1}(X, \mathbb{R})$ is equipped with a quadratic form of signature $(+, -, \ldots, -)$ (the Beauville-Bogomolov form). An automorphism $f \in \mathrm{Aut}(X)$ is said to be parabolic if it induces a parabolic isometry f^* on the projectivized cone of positive-square elements, viewed as a model for the hyperbolic space. This amounts to say that f^* fixes an integral nef isotropic class h, and is not of finite order. A famous conjecture, which is verified in all known cases, implies that X admits a lagrangian fibration $p : X \to B$, and h is the pullback of an ample class from the base. We shall assume that this indeed holds.

In the first lecture I shall explain some basics on parabolic automorphisms, in particular Lo Bianco's theorem that the induced automorphism f_B of B is of finite order, so that a certain power of f acts on the fibers. I also shall explain why, possible after taking a further power, f is a fiberwise translation.

In the second lecture I plan to prove that the orbits of f are dense (in the euclidean topology) in a general fiber of p, after a joint work with M. Verbitsky. This is a consequence of Hodge theory. I shall also remark that Bakker's recent proof of Matsushita's conjecture essentially reduces the problem to several known results about the so-called Betti map, by André-Corvaja-Zannier-Gao and Voisin, at least in the projective case. This approach yields more precise results, for example the existence of a dense set of finite orbits.

Finally, in the third lecture I shall focus on a recent joint work with S. Cantat, where we provide a "dynamical" proof of maximality of the rank of the Betti map in the hyperkähler case. I shall also explain how the non-projective case reduces to the projective one by using degenerate twistor deformations studied by Soldatenkov-Verbitsky.

Yoonjoo Kim

Title (Lecture I): *Néron models over curves*

Abstract: The goal of these three talks is to study the behavior of Lagrangian fibrations of smooth symplectic varieties (e.g., hyper-Kahler manifolds) using the so-called Néron model theory. Every smooth family of abelian varieties over an an open curve C_0 uniquely extends to a smooth family of commutative Lie groups over its compactification C; this extension is called the Néron model. Since its introduction in the 1960s, Néron model theory has proven to be a powerful tool in studying 1-parameter families of abelian varieties. The first talk will provide an overview of this theory, focusing on its three different construction methods. The first method is Néron's original approach, while the other two are more suitable for future generalizations to higher-dimensional bases. Minimal elliptic surfaces (=Lagrangian fibrations over curves) can be analyzed using Néron models; this was the original motivation of Néron.

Title (Lecture II): *Néron models over higher-dimensional bases and Lagrangian fibrations*

Abstract: For a long time, Néron model theory was primarily used over 1-dimensional bases. In 2019, Holmes attempted to generalize the definition of Néron models to higher-dimensional bases. Néron's original construction method no longer works over higher-dimensional bases; in fact, Holmes observed that the Néron model frequently fails to exist. In the second talk, I will explain the main theorem that Holmes's generalized Néron model does exist in the case of Lagrangian fibrations. This generalizes Néron's result of minimal elliptic fibrations to higher-dimensional Lagrangian fibrations. The construction utilizes two group objects that are interesting in their own right: the automorphism group scheme and Picard group scheme over higher-dimensional bases. Understanding the birational behaviors of Lagrangian fibrations is the key to constructing the Néron model.

Title (Lecture III): *Using the Néron model to understand Lagrangian fibrations*

Abstract: One interpretation of the Néron model is the moduli space of certain special birational automorphisms of a Lagrangian fibration. This perspective allows us to transfer the properties of the Néron model to those of Lagrangian fibrations. The goal of the third talk is to see some applications of the existence of the Neron model from this viewpoint. First, we can consider a Lagrangian fibration as a minimal model-compactification of a smooth commutative group scheme-torsor. Second, Ngô's techniques

for Hitchin fibrations can be extended to many Lagrangian fibrations through the Néron model. Third, it gives an alternative way to approach Hwang-Oguiso's classification theorem of the general singular fibers in Lagrangian fibrations. Finally, the notion of the Tate-Shafarevich twist can be understood via the Néron model.

Zhiyuan Li

Title (Lecture I–III): *Filtrations on Chow Groups of Hyperkähler Varieties*

Abstract: This series investigates the rich interplay between hyperkähler geometry and filtrations on Chow groups—a nexus bridging Hodge theory, derived categories, and motivic frameworks.

Lecture I begins with an overview of foundational filtrations, notably the conjectural *Bloch-Beilinson filtration* and the *Beauville-Voisin (BV) filtration*. These stratify Chow groups in alignment with Hodge-theoretic and cohomological structures, drawing inspiration from the celebrated Beauville-Voisin decomposition for $K3$ surfaces, where a canonical zero-cycle governs the structure of CH_0.

Lectures II–III will focus on the case of Bridgeland moduli spaces. We present recent progress in constructing BV-type filtrations on these spaces, leveraging techniques from derived algebraic geometry and stability conditions. Key examples, open conjectures, and implications for the Beauville splitting conjecture will be discussed, alongside emerging problems in the study of other hyperkähler varieties.

Keiji Oguiso

Title (Lecture I–III): *Birational/biregular automorphisms of higher dimensional smooth projective varieties with trivial canonical class via algebraic dynamics—Primitivity and existence of Zariski dense orbit*

Abstract: I would like to discuss birational/biregular automorphisms, mostly of "positive entropy", of higher dimensional smooth projective varieties with trivial canonical class via algebraic dynamics, especially, primitivity and existence of Zariski dense orbit, mainly over the complex number field $\mathbb{C}$ and possibly over an algebraic closure $\overline{\mathbb{Q}}$ of the field of rational numbers $\mathbb{Q}$. In the course of talks, I also would like to present several concrete examples of strict Calabi-Yau manifolds with primitive birational/biregular automorphisms defined not only over $\mathbb{C}$ but also over $\overline{\mathbb{Q}}$. Some of them are constructed/studied by using hyperkähler manifolds.

Tentative plan of my talk is as follows:

Lecture I:

(1) Amerik-Campana's rational fibration (after Amerik-Campana and Bell-Ghioca-Reichstein)

(2) Target varieties with primitive birational automorphisms and some relation with dynamical degrees with some explicit examples from abelian varieties (after De-Qi Zhang, Truong-myself, and Chen-Lin-myself)

Lecture II:

(1) Varieties fibered by varieties of general type and orbit of equivariant birational automorphism (after Lo Bianco with a slight modification)

(2) Several concrete examples of higher dimensional smooth projective varieties with trivial canonical class with primitive birational/biregular automorphisms (after Lo Bianco and myself)

Lecture III:

(1) Brief review of classification of Calabi-Yau threefolds with c_2-contractions (after Sakurai-myself)

(2) Calabi-Yau threefolds with a biregular automorphism with Zariski dense orbit (work in progress)