Progress in Complex Geometry

April 21–24, 2025

Speaker: **Damian Brotbek** (U. Lorraine)

Title: *Hyperbolicity and jet differentials*

Abstact: A classical theorem of Picard states that an entire function on the complex plane omits at most one point. Equivalently, it says that there is no non-constant holomorphic map from the complex plane into the complement of three distinct points in the projective line. Generalizing this property, given a complex manifold X, we say that a X is Brody-hyperbolic if there exists no non-constant map from the complex plane into X (so called entire curves). It is conjectured that the hyperbolicity properties of X are closely related to the geometry of X by means of the positivity properties of the canonical bundle. In this lecture, I will give an introduction to this area of research, the main conjectures of the field. I will also give an introduction to the theory of jet differentials, which is a powerful tool to study the entire curves contained in X. A particular emphasize will be put on the relations between the analytic and the algebraic aspects of the theory.

Speaker: **Young-Jun Choi** (Pusan National U.)

Title: *A Characterization of the Ball via the Kähler-Einstein Potential and Towards Bounded Symmetric Domains*

Abstact: Characterizing the universal coverings of negatively curved compact Kähler manifolds is a longstanding problem in complex geometry. Wong and Rosay proved that if the universal covering is a bounded smooth domain, then it must be biholomorphic to the unit ball. On the other hand, Frankel showed that if the universal covering is a bounded convex domain, then it is necessarily a bounded symmetric domain. Both results rely on the scaling method, a powerful technique that is not easily applicable to general complete Kähler manifolds.

In this talk, we will discuss a characterization of the unit ball via the Kähler-Einstein potential and consider how this approach can be extended to bounded symmetric domains. This is based on joint work with Kang-Hyurk Lee and Aeryeong Seo.

Speaker: **Bin Guo** (Rutgers U.)

Title: *Complex Monge-Ampère equations in Kähler geometry*

Abstact: We will discuss the role of complex Monge-Ampère equations as auxiliary equations in deriving sharp analytic and geometric estimates in Kähler geometry. Two applications will be given: the first one is the C^0 estimate for a class of fully nonlinear PDE's on complex manifolds; and the second one is the exploration of Green's functions and their geometric applications on Kähler manifolds.

Speaker: **Jie Liu** (AMSS, CAS)

Title: *Kawamata—Miyaoka inequality for $\mathbb{Q}$-Fano varieties*

Abstact: The classical Bogomolov—Miyaoka–Yau inequality says that the second Chern class of a canonically polarized projective variety with mild singularities can be bounded from below by its volume. I will present an analogous inequality, called the Kawamata—Miyaoka inequality, for Fano varieties with Picard number one and then I will discuss its applications in the explicit classification of Fano threefolds with Picard number one. This is based on joint works with Haidong Liu and also joint work with Chen Jiang and Haidong Liu.

Speaker: **Shin-ichi Matsumura** (Tohoku U.)

Title: *Singular Nakano positivity of direct image sheaves*

Abstact: Two notions of positivity, Nakano positivity and Griffiths positivity, exist for the Chern curvature of vector bundles. The theory of Griffiths positivity for singular metrics is already well-developed, and via its connection to the L^2-extension theorem with optimal estimates, the positivity of direct image sheaves of relative canonical bundles, which is particularly important for applications in algebraic geometry, has been established. In contrast, the theory of Nakano positivity for singular metrics is still under development. In this talk, I will introduce several aspects of Nakano positivity for singular metrics and explain my recent work on singular Nakano positivity for direct image sheaves. This work is based on joint work with Inayama (Tokyo University of Science) and Watanabe (Chuo University).

Speaker: **Shigeharu Takayama** (U. Tokyo)

Title: *Analytic approach to Kawamata's semi-positivity and a canonical bundle formula*

Abstact: The canonical bundle formula is a key device in the classification theory of algebraic varieties to study fiber spaces. The original form is given by Kodaira for elliptic fibrations. The formula, in general, relates canonical bundles of two projective varieties X and Y with a morphism $f : X \to Y$

whose general fiber has "trivial" canonical bundle. We would expect to having $K_X = f^*(K_Y+M+B)$ with certain $\mathbb{Q}$-divisors M and B on Y, which respectively reflects a variation of complex structures of fibers and singularities of the map f. As a general expectation/speculation, the "moduli part" M should be "abundant". The "boundary part" B is related to monodromy around the singular fiber and described by "log-canonical threshold" of the fiber. We need actually more technical generalizations of such formulas for various applications, that is we would like to discuss. We will explain backgrounds, necessarily materials, basic constructions, and key technical inputs.

Speaker: **Dror Varolin** (Stony Brook U.)

Title: *Interpolation and Sampling in Bergman Spaces*

Abstact: Motivated by problems from solid state physics and signal analysis, a theory of interpolation and sampling was initiated in the early 1990s that was analogous to a much earlier theory developed by Beurling in the 1930s and later by Shannon in the 1950s. Early results by Seip, Berndtsson, Ortega Cerdà and others provided a rather complete mathematical story for interpolation and sampling in the Fock space of entire functions, the Bergman space of the unit disk, and their weighted analogues. Some work was done in Bergman spaces on Riemann surfaces. There has also been some work in higher dimensions, but in the higher dimensional setting relatively little is known, and there remain many natural problems. In this talk I will survey the theory, present some new results, and propose a number of open problems and some new directions.

Speaker: **Juanyong Wang** (AMSS, CAS)

Title: *An abundance-type result for the tangent bundles of compact Kähler manifolds*

Abstact: The Abundance conjecture is a central problem in algebraic geometry and complex geometry. It asserts that a weak positivity condition (nefness, pseudoeffectivity) for the canonical bundle implies a strict stronger one (semiampleness, effectivity). This abundance principle is expected to hold for other canonically defined vector bundles on compact Kähler manifolds. For the case of tangent bundles, this is in fact a conjecture of Campana-Peternell (1991), which states that if the tangent bundle of a compact Kähler manifold is nef then it is globally generated up to finite étale cover. In this talk, I will explain our recent progress towards the Campana-Peternell conjecture: we prove that the tangent bundle is big thus semiample.

Speaker: **David Witt Nyström** (Chalmers U.)

Title: *On the Monge-Ampère equation for test configurations*

Abstact: A central problem in complex geometry is to determine whether a compact Kähler manifold X admits a Kähler metric with constant scalar curvature. According to the Yau-Tian-Donaldson (YTD) conjecture there should exist such a metric in a given Kähler class alpha if and only if the pair (X, α) is K-stable. Here K-stability is defined using so-called test configurations, which are certain $\mathbb{C}^*$-equivariant degenerations of (X, α) fibered over $\mathbb{P}^1$.

The YTD-conjecture was famously proved in the Fano case by Chen-Donaldson-Sun. An alternative proof using non-Archimedean (nA) pluripotential theory was later given by Berman-Boucksom-Jonsson. In nA pluripotential theory test configurations play the role of nA Kähler metrics. In particular a test configuration has an associated nA Monge-Ampère measure, and a crucial result of Boucksom-Favre-Jonsson ensures the solvability of the corresponding nA Monge-Ampère equation. Note that this is a nA analogue of Yau's theorem.

In my two lectures I will discuss this nA Monge-Ampère equation and how its solvability is related to the theory of big cohomology classes. This might sound technical and complicated, but I hope to show that in fact it is quite natural. We will also see that the nA analogue of Yau's theorem is true in the general Kähler setting, and not only in the algebraic setting considered by Boucksom-Favre-Jonsson. This last part is based on joint work in progress with Pietro Mesquita Piccione.

Speaker: **Ming Xiao** (U.C. San Diego)

Title: *Rationality and algebraicity of the Bergman kernel*

Abstact: The Bergman kernel is the reproducing kernel of the Bergman space, consisting of square-integrable holomorphic functions on a domain. It encodes important geometric and analytic properties of the domain, and plays a key role in complex analysis. In this talk, we first present some preliminaries about the Bergman kernel. Then we will discuss some recent progress on characterizing special domains in terms of algebraic properties of the Bergman kernel. This talk is based on our joint work with Ebenfelt and Xu.

Speaker: **Junyi Xie** (Peking U.)

Title: *Algebraic dynamics and recursive inequalities*

Abstact: We get three basic results in algebraic dynamics: (1) We give the first algorithm to compute the dynamical degrees to arbitrary precision. (2) We prove that for a family of dominant rational self-maps, the dynamical degrees are lower semi-continuous with respect to the Zariski topology. This implies a conjecture of Call and Silverman. (3) We prove that the set of periodic points of a cohomologically hyperbolic rational self-map is Zariski dense.

In fact, for every dominant rational self-map, we find a family of recursive inequalities of some dynamically meaningful cycles. Our proofs are based on these inequalities.