Workshop on Hessenberg varieties,
Lusztig varieties, and affine Grassmannians

June 30 –July 4, 2025

Speaker: **Alex Abreu** (Universidade Federal Fluminense)

Title: Parabolic Lusztig Varieties

Abstract: This lecture series explores the geometry and representation theory of parabolic Lusztig varieties, which arise as analogues of classical Lusztig varieties associated with partial flag varieties. Particular emphasis will be placed on the geometry of their intersection cohomology and the local systems arising from their decomposition. The series aims to highlight the role of parabolic analogues in phenomena such as the monodromy of character sheaves, modular representations, and positivity conjectures.

Speaker: **Pramod Achar** (Louisiana State University)

Title: An introduction to affine Grassmannians

Abstract: The goal of these lectures will be to start from scratch and learn what the affine Grassmannian of a reductive group is, and how its geometry is related to the representation theory of reductive groups. I will emphasize the examples of GL_n and SL_2 throughout. Here is a rough outline of the three lectures:
Lecture 1: definitions; lattice model for GL_n; orbits
Lecture 2: perverse sheaves and Lusztig's q-analogue of the weight multiplicity
Lecture 3: convolution product and the geometric Satake equivalence

Speaker: **Hiraku Abe** (Okayama University of Science)

Title: On Peterson isomorphism and toric geometry

Abstract: Dale Peterson discovered that there is an isomorphism between (localization of) the quantum cohomology ring of the flag variety and (localization of) the homology ring of the affine Grassmannian. What he observed is that this isomorphism is naturally encoded in the geometry of a distinguished subvariety of the flag variety, which is now called Peterson variety.

In this talk, I will explain that the "localizing functions" in Peterson isomorphism provide a connection between Peterson variety and toric geometry. I am not an expert on Peterson isomorphism, and there will be no new results in my talk, but I wish to discuss this phenomenon with participants.

Speaker: **Donggun Lee** (IBS-CCG)

Title: Geometry of regular semisimple Lusztig varieties

Abstract: Lusztig varieties can be seen as variants of both Schubert varieties and Hessenberg varieties. For example, they admit a Weyl group action on their intersection cohomology through monodromy, and their singularities can be resolved via Bott-Samelson-type resolutions.

I will present results on their geometry, including vanishing theorems for the cohomology of line bundles, their relations to Hessenberg varieties, and diffeomorphism types. Along the way, we also establish that their open cells are affine, and that the same is true for Deligne-Lusztig varieties, settling a question that has been open since the foundational paper of Deligne and Lusztig. Some of the results were motivated by our forthcoming study of the representations via a trace map of the Hecke algebra. Based on joint works with Patrick Brosnan and Jaehyun Hong.